How to accurately measure irregular 2-D surface areas
(without a computer)

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[Adapted from an article that I found in *Scientific American* magazine, August, 1958, p. 107]

Occasionally, one needs to accurately measure the area of a flat surface that has an irregular boundary (Fig. 1). This is not always an easy task. One pre-computer-age method involved placing a transparency with grid lines onto the surface and then counting the number of squares (and the various innumerable fractions of incomplete squares) within the object's boundary. When measuring large areas, this method can be *extremely* tedious, the areas within the incompletely covered squares are always rough estimates, and worse, it is very easy to lose count half-way through the measurement.

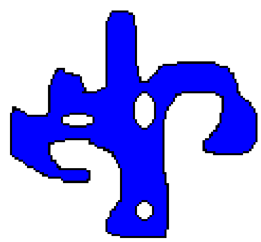

Fig. 1. A surface area such as this would be a challenge to measure by a manual (grid) method..

Another method (and the one that is most commonly in use today) involves scanning an image of the surface into a computer imaging program such as AutoCad^{Tm}, NIH Image, or Scion, and then let the program compute its area. But there is also a low-tech way to measure irregularly-shaped areas and it is very accurate. To employ this method, all that you will need is a laboratory-quality scale with reasonably high precision, one that can measure weight to at least 0.001 gram (1 milligram). The best scales for this purpose are the high quality electronic scales that have large pans.

The procedure is relatively straightforward:

Using a sharp pencil, make an accurate trace of the surface to be measured. In order for the “weight method” to work, the thickness of the tracing medium must be *extremely* uniform. Regular tracing paper works well. Another alternative is to use clear Mylar^{Tm}. The thickness of Mylar^{Tm} (and, therefore, its density) is extremely uniform. For measuring small areas, Mylar^{Tm} is less versatile. Mylar^{Tm} is stiff, and it is harder to cut out small shapes than it is if you are using paper.

Next, use a good pair of scissors, or, if you have a steady hand, an X-acto^{Tm} knife, to *carefully* cut out the trace of the irregular area. Lay the cut out area aside.

Using the SAME tracing medium from which you cut out the trace of the surface, carefully cut out a square of *exactly* 1 square centimeter (or, if you are using English units, exactly 1 square inch). This square piece of paper will be your *standard*. Next, weigh both the square and the cut out trace of the irregular area (if necessary, crumple the paper so that it fits on the scale). If the traced area is *still* too large to fit in the scale's pan (or if the entire trace is too heavy for the scale), cut it into smaller fragments and weigh each piece separately, then add up the individual weights to give you the total weight of the irregular area.

Since you know the weight of 1 cm^2 of tracing paper, you also know the relationship between its weight and its surface area.

This relationship can be written as:

$$\frac{1\ \text{cm}^2\ \text{of paper}}{\text{weight of } 1\ \text{cm}^2\ \text{of paper}} = \frac{\text{Unknown area of irregular shape (in cm}^2\text{)}}{\text{weight of traced irregular surface}}$$

Solving for "Unknown area", we get:

$$\text{Unknown area (in cm}^2\text{)} = \frac{\text{weight of traced surface} \times 1\ \text{cm}^2}{\text{weight of } 1\ \text{cm}^2\ \text{of tracing paper}}$$

ERROR ANALYSIS

In order to determine how accurate the "weight method" is, I conducted an error analysis. Instead of cutting out an area of irregular shape, I cut out a rectangle whose area was accurately known. I found that the area calculated by the "weight method" differed from the true area by only 0.6%. This tiny error might be reduced even further had I instead used a better quality tracing medium (I used the only thing I had handy at the moment, newspaper) and a more precise laboratory scale (I used a scale for reloading rifle ammunition). Results obtained from using fancy CAD software might be a bit more accurate than the "weight method" described here, but I suspect that the difference would not be large.